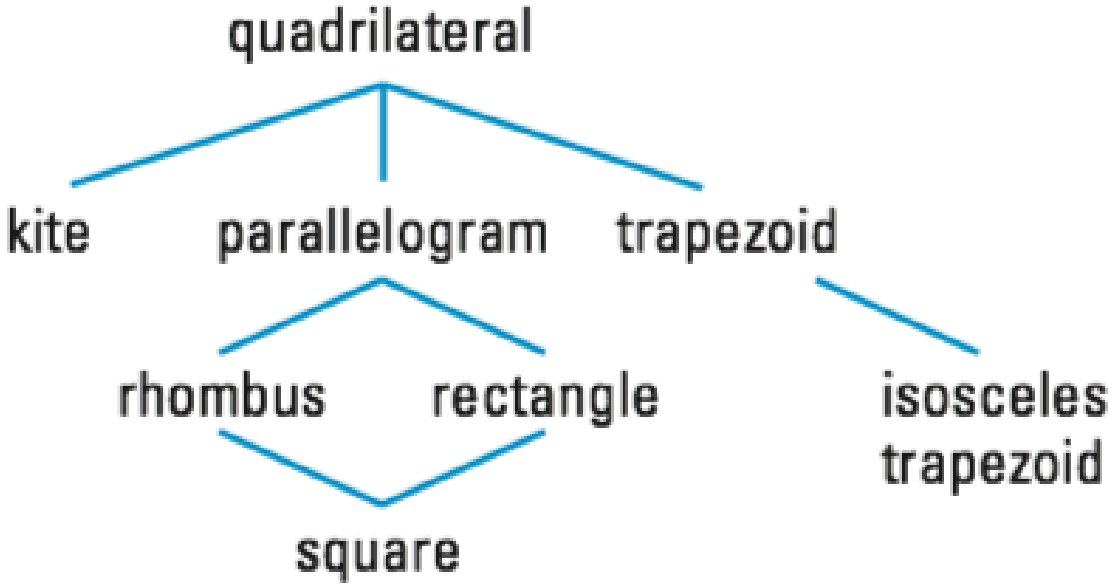
Chapter 6

Quadrilaterals

Section 6
Special Quadrilaterals

GOAL 1: Summarizing Properties of Quadrilaterals

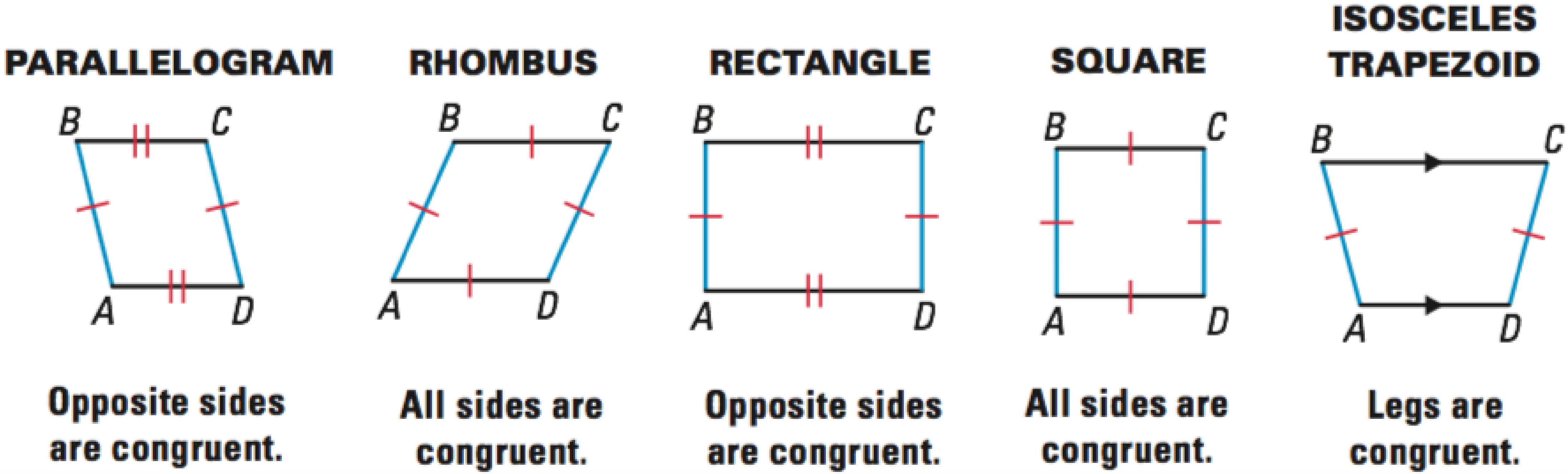
In this chapter, you have studied the seven special types of quadrilaterals at the right. Notice that each shape has all the properties of the shapes linked above it. For instance, squares have the properties of rhombuses, rectangles, parallelograms, and quadrilaterals.



Example 1: Identifying Quadrilaterals

1 or more

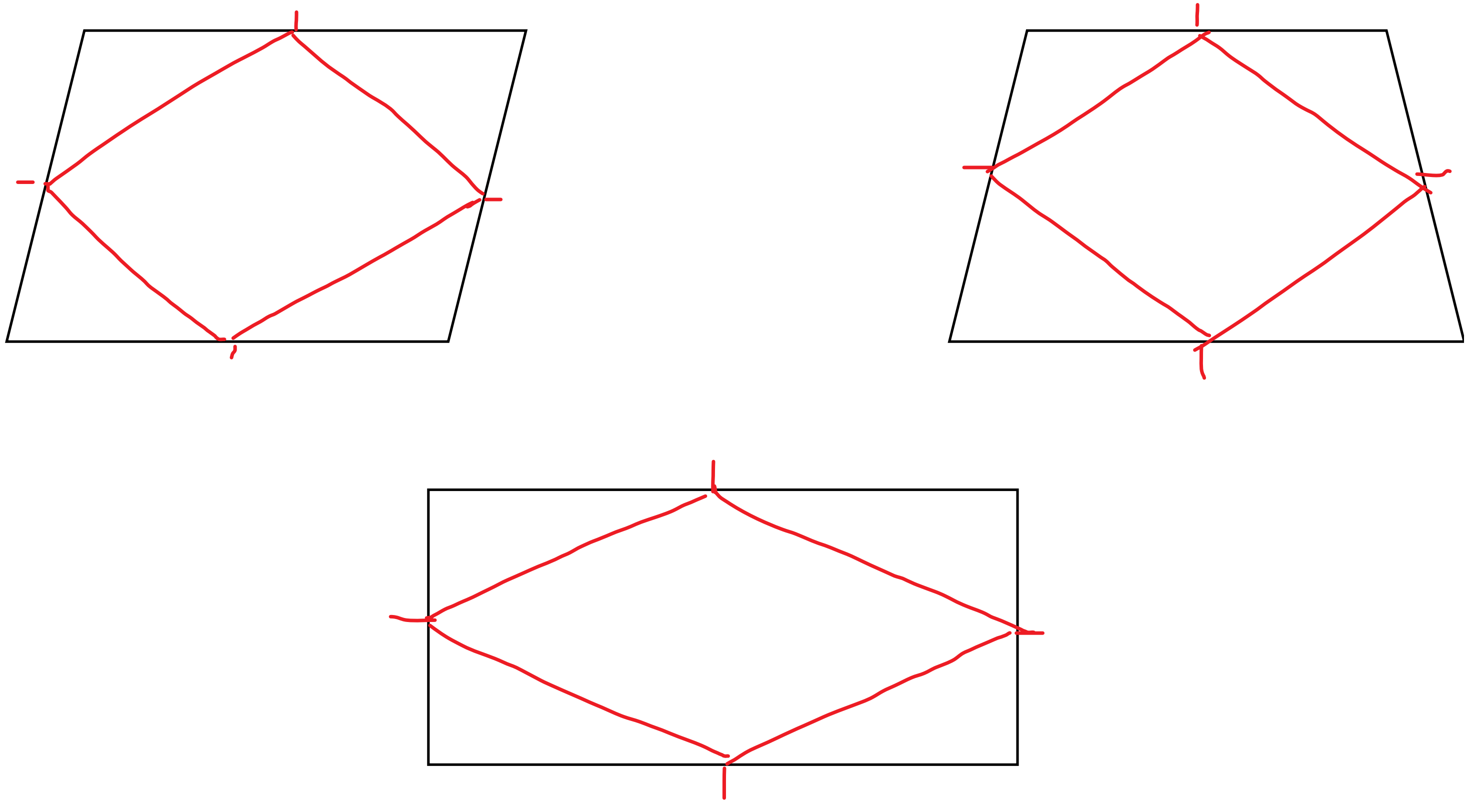
Quadrilateral ABCD has at least one pair of opposite sides congruent. What kinds of quadrilaterals meet this condition?



Example 2: Connecting Midpoints of Sides

When you join the midpoints of the sides of any quadrilateral, what special quadrilateral is formed?

parallelogram



GOAL 2: Proof with Special Quadrilaterals

When you want to prove that a quadrilateral has a specific shape, you can use either the definition of the shape as in Example 2, or you can use a theorem.

CONCEPT SUMMARY — **PROVING QUADRILATERALS ARE RHOMBUSES**

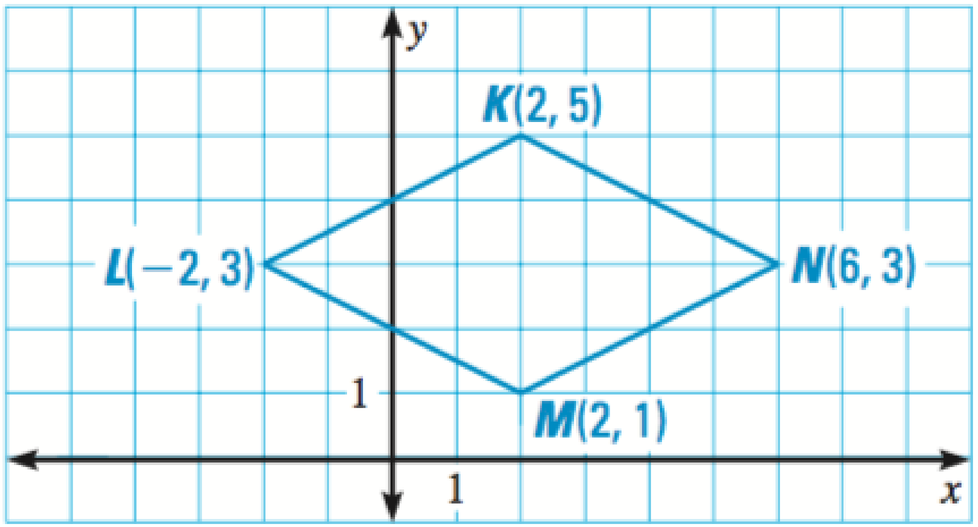
You have learned three ways to prove that a quadrilateral is a rhombus.

1. You can use the definition and show that the quadrilateral is a *parallelogram* that has four congruent sides. It is easier, however, to use the Rhombus Corollary and simply show that all four sides of the quadrilateral are congruent.

2. Show that the quadrilateral is a parallelogram *and* that the diagonals are perpendicular. (*Theorem 6.11*)

3. Show that the quadrilateral is a parallelogram *and* that each diagonal bisects a pair of opposite angles. (*Theorem 6.12*)

Example 3: Proving a Quadrilateral is a Rhombus



Show that KLMN is a rhombus.

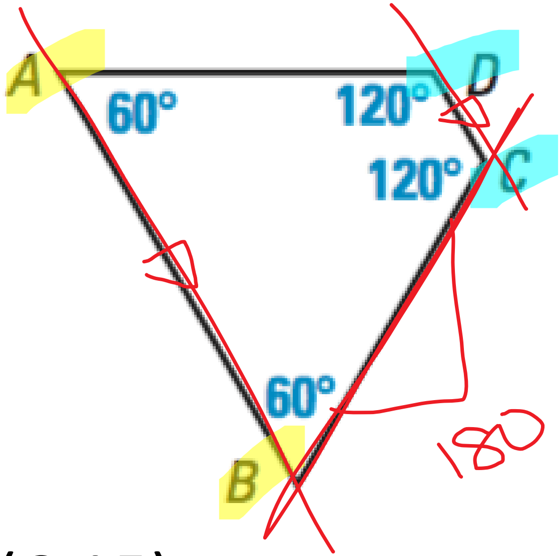
LK $\to \sqrt{(-2-2)^2+(3-5)^2} \to \sqrt{16+4} \to \sqrt{20}$

KN $\to \sqrt{(6-2)^2+(3-5)^2} \to \sqrt{16+4} \to \sqrt{20}$

ML $\to \sqrt{(2-{}^-2)^2+(1-3)^2} \to \sqrt{16+4} \to \sqrt{20}$

MN $\to \sqrt{(6-2)^2+(1-3)^2} \to \sqrt{16+4} \to \sqrt{20}$

Example 4: Identifying a Quadrilateral



What type of quadrilateral is ABCD? Explain your reasoning.

only 1 pair of parallel sides → trapezoid

base angles are congruent → isosceles trapezoid (6.15)

Example 5: Identifying a Quadrilateral

The diagonals of quadrilateral ABCD intersect at point N to produce four congruent segments: AN ≅ BN ≅ CN ≅ DN. What type of quadrilateral is ABCD? Prove that your answer is correct.

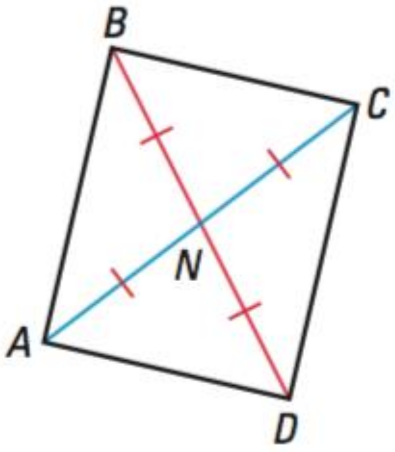
First prove that *ABCD* is a parallelogram.

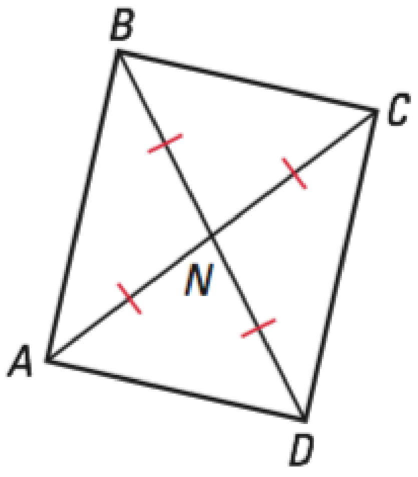
Because $\overline{BN} \cong \overline{DN}$ and $\overline{AN} \cong \overline{CN}$, $\overline{BD}$ and $\overline{AC}$ bisect each other. Because the diagonals of *ABCD* bisect each other, *ABCD* is a parallelogram.

Then prove that the diagonals of *ABCD* are congruent.

From the given you can write $BN = AN$ and $DN = CN$ so, by the Addition Property of Equality, $BN + DN = AN + CN$. By the Segment Addition Postulate, $BD = BN + DN$ and $AC = AN + CN$ so, by substitution, $BD = AC$.

So, $\overline{BD} \cong \overline{AC}$.





▶ *ABCD* is a parallelogram with congruent diagonals, so *ABCD* is a rectangle.

EXIT SLIP